ISSN: 1803-3814

TWO-DEGREE-OF-FREEDOM CONTROLLER TUNING

Stanislav Lang[1], Radomil Matoušek[1,2,✉], Ivan Švarc[1]

Brno University of Technology
[1]Institute of Automation and Computer Science
[2]Central European Institute of Technology (CEITEC)
Czech Republic
stanislav.lang@vutbr.cz, svarc@fme.vutbr.cz, rmatousek@vutbr.cz[✉]

Abstract: *Two-degree-of-freedom controllers have the ability to affect the dynamics of a system when the reference value changes. The answer to the question of parameter tuning for this additional filter still remains unclear we describe a new method for the design of said controller. We compare the behaviour of the controllers designed using the presented method versus the classic method on several instances.*

Keywords: *two degree of freedom controller (2DOF controller), PID controller, disturbance variable, require variable*

1 Introduction

Two Degree of Freedom Controllers (2DOF) are not the most used types of controllers, however, the corresponding models are readily available in Matlab Simulink. They differ from the classic PID controllers in an additional input filter for reference value setting [1]. Because of this filter, better dynamic properties are achieved, particularly when changing the reference value and dynamic of disturbance variable remains unchanged [2].

2 Basic Forms of Two-Degree-of-Freedom Controllers

We start the description from a simple control circuit with one-degree-of-freedom controller, see Fig. 1, with weights set on $b = c = 1$. In this case we obtain classic PID controller with one degree of freedom. The use of a two-degree-of-freedom controller (2DOF) is attained by changing the weights of the reference value for the proportional and the derivation part as shown in figure.

We see that with change of the disturbance variable $v(t)$ nothing changes, quality of control stays the same, and the weights b, c affect only changes of reference value $w(t)$. This is an important property of 2DOF controllers – reduced overshoot for reference value changes and weights b, c do not interfere with control when disturbance variable changes.

On the input of the integration part of the controller there (inevitably) must be an error variable $e(t)$ and, therefore, the weight of the integration part must be set to $a = 1$. In the case of a controller without an integration part, the error variable must be in the input for the proportional part with the weight set to $b = 1$.

The controller with 2DOF in Fig. 2 can be described [1] by the following equation

$$U(s) = K_p \left\{ [bW(s) - Y(s)] + \frac{1}{T_I s} [W(s) - Y(s)] + T_D s \, [cW(s) - Y(s)] \right\}. \tag{1}$$

This equation can be written in the form

$$U(s) = K_p \left(b + \frac{1}{T_I s} + cT_D s \right) W(s) - K_p \left(1 + \frac{1}{T_I s} + T_D s \right) Y(s) \tag{2}$$

$$U(s) = G_{ff}(s)W(s) - G_R(s)Y(s) \tag{3}$$

and equation (3) corresponds to scheme in Fig. 2. According to [1] substitute in (3)

$$G_{ff}(s) = G_F(s) - G_R(s), \tag{4}$$

where

$$G_F(s) = \frac{G_{ff}(s)}{G_R(s)} = \frac{cT_D T_I s^2 + bT_I s + 1}{T_D T_I s^2 + T_I s + 1} \tag{5}$$

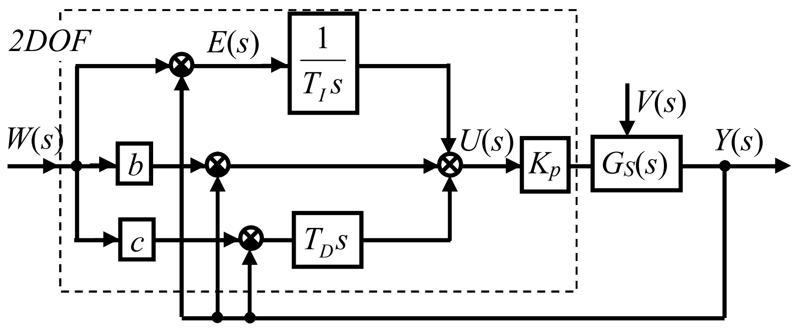


Figure 1: PID controller with two degrees of freedom 2DOF

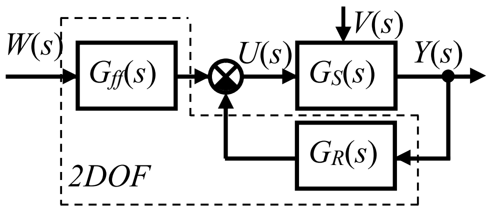


Figure 2: 2DOF controller corresponding to equation (3)

is the transfer of the input filter of the reference value w, resulting in the equation

$$U(s) = G_F(s)G_R(s)W(s) - G_R(s)Y(s) \tag{6}$$

and the corresponding scheme of the 2DOF controller in Fig. 3 as described in [1]. All mentioned structures of circuits with the 2DOF controller (and possibly some other forms) are equivalent to each other.

The usage and the design of controllers is the following: We search for the optimal parameters (in a similar heuristic manner as [3], [4], [5]) for classic PID controller for changes in the disturbance variable (quadratic control area, Ziegler-Nichols (Z-N)), which are standard in industrial applications [6]. With appropriate modeling, we can often observe that there is quite a big overshoot and a slow response to step changes of reference value (this kind of change is not so common for the disturbance variable – however, is does occur) [7]. Then we propose weights b, c for the 2DOF controller so the dynamics for reference value changes behaves better. The dynamics of changes in disturbance variable remains unchanged.

3 Two-Degree-of-Freedom Controller Tuning

The choice of weights b and c is one of the important aspects for the application of the 2DOF controllers. One option is a method described in [1] that uses input filter with transfer in the form (5). In product of this filter $G_F(s)$ with control transfer $G_R(s)$. The denominator of filters transfer, $G_S(s)$, can be compensated with numerator of control transfer and by the choice of suitable weights b and c. This method prefers the response time over the size of the overshoot.

The second option (and this is the main contribution of this paper) is the method of critical state whose principle lies in making the control circuit to oscillate with excluded integration ($T_I = \text{inf}$) and derivation part ($T_D = 0$), and changing controller's gain K_P. The circuit is brought into an oscillating state, similarly as in Ziegler-Nichols method. This way, the critical gain $K_{p\ krit}$ is determined, and from a periodic signal of the controlled variable y the critical period T_{krit} is determined. Experiments were conducted with higher number of control circuits of various character systems proportional, integration, with transport delay, etc. From the determined optimal values Table 1 was created for the optimal choice of weights b and c.

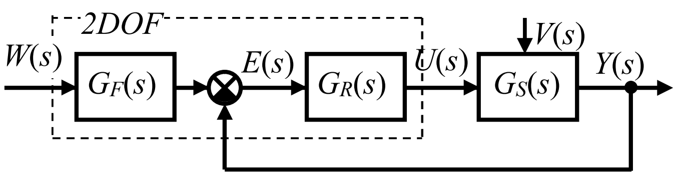


Figure 3: 2DOF controller corresponding to equation (6)

Table 1: Optimal weights of 2DOF controllers

$controller$	b_{opt}	c_{opt}
PI	$0.032K_{p\ krit}$	-
PID	$0.016K_{p\ krit}$	$0.05T_{krit}$

4 Example

Determine optimal settings of 2DOF controller PID for system with transfer

$$G_S(s) = \frac{6}{48s^3 + 44^2 + 12s + 1}. \tag{7}$$

Solution: According to Ziegler-Nichols method, the critical point of control system was determined $K_{p\ krit} = 1.67$; $T_{krit} = 13.29s$. Then by Z-N the transfer of PID controller for this system is

$$G_R(s) = 1.002\left(1 + \frac{1}{6.65s} + 1.6s\right). \tag{8}$$

If we use only the 1DOF controller, we can see in Fig. 4 that with the step change in the disturbance variable v the overshoot is 45% and response time $32s$. However with the step change in the reference value w the overshoot is 92% with the same response time. To eliminate the undesired overshoot and response for the change of the reference value we use the 2DOF controller and we set the weights according to Table 1.

$$b_{opt} = 0.627; c_{opt} = 0.05 \tag{9}$$

For changes in reference value w the overshoot is reduced to 3% with the response of $25s$ Fig. 4. Identical models with the same changes were realized for other systems and the results are shown in Table 2. From these (and many more) experiments we assembled Table 1 for the choice of optimal weights b, c for the 2DOF controllers.

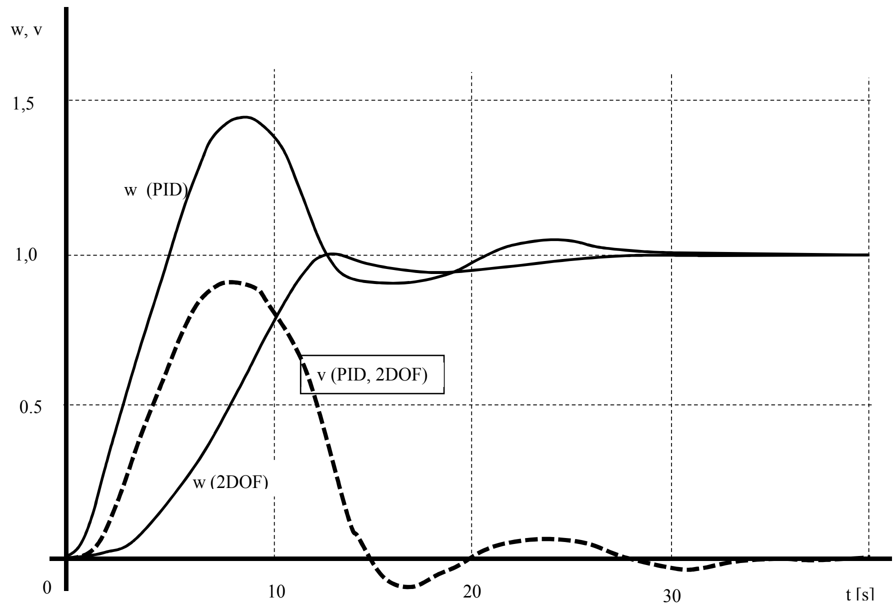


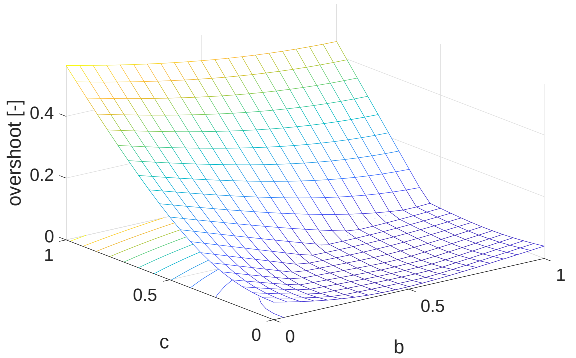
Figure 4: Comparison of overshoot and response for usage of 1DOF and 2DOF controllers

5 Conclusion

Theoretical reasoning and results of modeling imply the advantageousness of the use of the 2DOF controllers, because with the unchanged behavior for disturbance variable it significantly improves behavior for changes

Table 2: Optimal weights of 2DOF controllers

plant $G(s)$	kind of controller	transfer of optimal controller by Z-N	1DOF controller overshoot [%]	1DOF controller response [s]	2DOF controller weights b	2DOF controller weights c	2DOF controller overshoot [%]	2DOF controller response [s]
$\frac{6}{48s^3+44^2+12s+1}$	PID	$1.002\left(1+\frac{1}{6.65s}+1.6s\right)$	50	30	0.627	0.05	3	25
$\frac{1}{s(s+1)(s+2)}$	PI	$2.835\left(1+\frac{1}{3.7s}\right)$	80	51	0.2	-	1	51
	PID	$3.78\left(1+\frac{1}{2.225s}+0.534s\right)$	72	20	0.1	0.3	5	16
$\frac{1}{8s+1}e^{-s}$	PI	$4\left(1+\frac{1}{3.32s}\right)$	35	16	0.32	-	9	14
	PID	$6\left(1+\frac{1}{2s}+0.48s\right)$	65	22	0.16	0.2	15	20
$\frac{2}{(10s+1)(6s+1)(2s+1)^2}$	PI	$1.44\left(1+\frac{1}{20.169s}\right)$	50	170	0.1	-	0	160
	PID	$1.92\left(1+\frac{1}{12.15s}+2.916s\right)$	50	70	0.0512	1	0	50



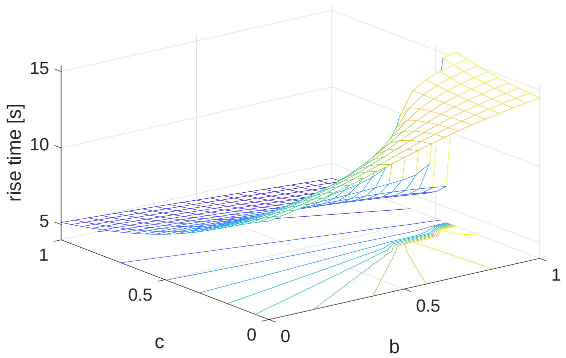


Figure 5: The parametric space for independent variables b, c given by quality of control criteria Overshoot (left) and Response (right) for the system $G_s(s)$ given by (7)

in reference value the undesirably big overshoots of control variable are reduced or even eliminated and the transition is faster. The main contribution is the method for the optimal choice (Fig. 5) of weights b, c for the 2DOF controllers (Table 1).

Acknowledgement: The author was supported by the Czech Science Foundation under the Project 16-08549S. This work was realized in Central European Institute of Technology (CEITEC) with research infrastructure supported by a project financed from European Regional Development Fund (Project CZ.1.05/1.1.00/02.0068).

References

[1] Viteckova, M., Vitecek, A.: 2DOF controller tuning. International Journal of Engineering Research in Africa 18, 57–64 (2015)
[2] Levine, W.S.: *The Control Handbook*, RC Press, Inc., Boca Raton (1996)
[3] Matousek, R., Popela, P., Kudela, J.: Heuristic approaches to stochastic quadratic assignment problem: Var and cvar cases. Mendel **23**(1), 73–78 (2017)
[4] Hanna, G., Miskarik, K., Dobrovsky, L., Osmera, P.: An efficient two-level optimization method for optimal tuning of controllers. Mendel **21**(1) 71–76 (2015)
[5] Navrat, A., Kubiena, J.: Control of the trident square robot. Mendel **21**(1), 115–120 (2015)
[6] Sekaj, I., Tarnik, M., Gora, R.: Genetic algorithm-based optimisation of blood glucose controller. Mendel **21**(1), 13–18 (2015)
[7] Marada, T.: PID controler parameters settings based on genetic algorithm for inverted pendulum purpose stabilization. Mendel **22**(1), 31–38 (2016)