

USING COMPLEX NETWORK VISUALIZATION AND ANALYSIS FOR UNCOVERING THE INNER DYNAMICS OF PSO ALGORITHM

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Abstract: In this study, we construct a complex network from the inner dynamic of Particle Swarm Optimization algorithm. The subsequent analysis of the network promises to provide useful information for better understanding the dynamic of the swarm that is not acquirable by other means. We present several network visualizations and numerical analysis. We discuss the observations and propose further directions for the research.

Keywords: Swarm Intelligence, Particle Swarm Optimization, Complex Network, Metaheuristic

1 Introduction

The Particle Swarm Optimization (PSO) [1-4] is a very popular metaheuristic for global optimization. The method is widely used in all areas of industrial optimization and remains in the center of interest of the research community. Despite several examples of excellent research [5, 6], many details about the inner dynamic of the algorithm remain hidden. Uncovering the inner rules of the swarming behavior might lead to advances in the design of more powerful variants of the basic method.

Recently the interconnection between metaheuristics and complex networks (CN) has been studied [7 – 10] and successfully applied to improve the performance of the algorithm [11]. The complex networks provide a promising tool for visualization and analysis of inner dynamics of evolutionary computational techniques (ECT) such as the PSO algorithm.

In this study, we construct a network structure from the inner dynamic of the PSO algorithm and investigate various aspects of such networks in order to propose possible direction for future research of interconnections between PSO and CN.

The main goals of this study are following:

- 1) Transform the inner dynamic of PSO algorithm into a network structure.
- 2) Analyze the network structure by numerical measures.
- 3) Compare the network statistics of PSO optimizing different fitness landscapes.
- 4) Identify and highlight the potentially most useful information from the network analysis.
- 5) Propose future directions for using the information from network analysis to enhance the performance of the PSO algorithm.

The rest of the paper is structured as follows: In section two, the PSO algorithm is described. In section three, the construction of the network is described. The experimental details are given in section four. The results are presented in the next section, followed by discussion and the conclusion.

2 Particle Swarm Optimization (PSO)

Original PSO [1] takes the inspiration from the flocking behavior of birds. A population (swarm) of candidate solutions (particles) of the optimization problem (defined by cost function) is randomly generated. Each particle is evaluated (assigned a quality quantification using the cost function). Next, the particles simulate a bird flight over the fitness landscape. The knowledge of global best found solution (typically noted gBest) is shared among the particles in the swarm. Furthermore, each particle has the knowledge of its own (personal) best found solution (noted pBest). Last

important part of the algorithm is the velocity of each particle that is taken into account during the calculation of the particle movement. The new position of each particle is then given by (1), where \vec{x}_i^{t+1} is the new particle position; \vec{x}_i^t refers to current particle position and \vec{v}_i^{t+1} is the new velocity of the particle.

$$\vec{x}_i^{t+1} = \vec{x}_i^t + \vec{v}_i^{t+1} \quad (1)$$

To calculate the new velocity the distance from \overrightarrow{pBest} and \overrightarrow{gBest} is taken into account alongside with current velocity (2).

$$v_{ij}^{t+1} = w \cdot v_{ij}^t + c_1 \cdot Rand \cdot (pBest_{ij} - x_{ij}^t) + c_2 \cdot Rand \cdot (gBest_j - x_{ij}^t) \quad (2)$$

Where:

v_{ij}^{t+1} - New velocity of the i th particle in iteration $t+1$. (component j of the dimension D).

w - Inertia weight value. v_{ij}^t - Current velocity of the i th particle in iteration t . (component j of the dimension D).

c_1, c_2 - Acceleration constants.

$pBest_{ij}$ - Local (personal) best solution found by the i th particle. (component j of the dimension D).

$gBest_j$ - Best solution found in a population. (component j of the dimension D).

x_{ij}^t - Current position of the i th particle (component j of the dimension D) in iteration t .

$Rand$ - Pseudo random number, interval (0, 1).

3 Network Construction

In order to construct a network structure from the inner dynamic of the PSO algorithm, the communication in the swarm is observed. The communication in the PSO is realized by the shared knowledge of global best solution ($gBest$).

A single node in the network represents a single particle alongside with the current iteration code. Therefore the theoretical maximal number of nodes in the network is the number of particles in the population times the number of the iterations of the algorithm. An edge is created between two nodes if a particle improved its $pBest$. In such situation, the connection is created between the nodes representing the particle in the current iteration and the same particle in the last iteration it has updated its $pBest$. Further, a link is created to the node representing particle that discovered the current $gBest$.

4 Experiment

In the following experiments, four well known benchmark functions for metaheuristic optimizers were used [12, 13].

A typical setting for the PSO algorithm has been used in this study. The population size (NP) was set to 20 and the number of iterations was set to 1000. The inertia weight w was set to linear decrease from 0.9 to 0.4 and acceleration constants c_1 and c_2 were set to 2. The dimensionality of the problem was set to 30. In the following subsection, we present several network visualizations with highlights.

As some of the network statistics presented in the results section are not commonly used in the ECTs community, we provide a brief description here:

A betweenness centrality is a measure of the centrality of a node in a network based on the number of shortest paths that pass through it. Betweenness centrality, therefore, identifies nodes in the network that are crucial for information flow. [14]

Degree centrality is a measure of the centrality of a node in a network and is defined as the number of edges (including self-loops) that lead into or out of the node. Degree centralities, therefore, lie between 0 and $n-1$ inclusive, where n is the number of vertices in a graph, and identify nodes in the network by their influence on other nodes in their immediate neighborhood. [14]

A closeness centrality is a measure of the centrality of a node in a network based on the mean length of all shortest paths from that node to every other reachable node in the network. Closeness centrality, therefore, identifies nodes in the network that are crucial for the quick spread of information. [14]

Graph density is the ratio of the number of edges divided by the number of edges of a complete graph with the same number of vertices. [14]

4.1 Sphere function

The sphere function represents the simplest unimodal optimization task. Figure 1 presents the visualization of the network constructed according to the rules described in Section 3 with the shortest path from first to the last node highlighted. The nodes and links belonging to different particles are distinguished by different colors in Figure 2. Finally, in Figure 3, the growth of the network in relation to the iterations of the algorithm is highlighted. The first 20% of iterations are represented by red color, magenta represents the 20-40% of iterations, green is the 40-60%, 60-80% is represented by

yellow color and last 80-100% of iterations is represented as cyan). Finally, several network statistics are presented in Table 1.

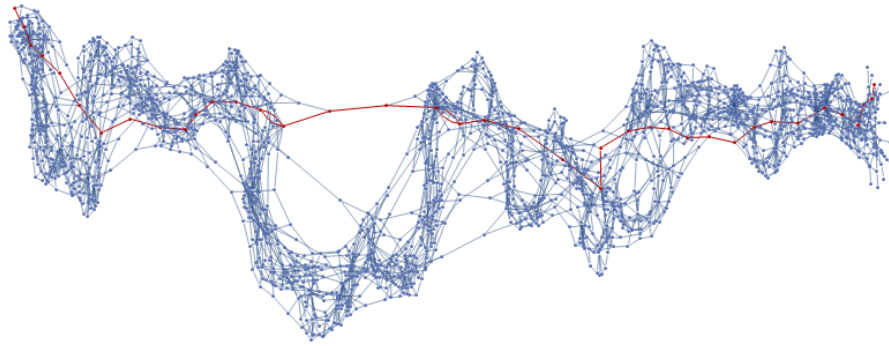


Fig. 1. Network visualization – Sphere function – Shortest path highlighted



Fig. 2. Network visualization – Sphere function – Particles paths in network

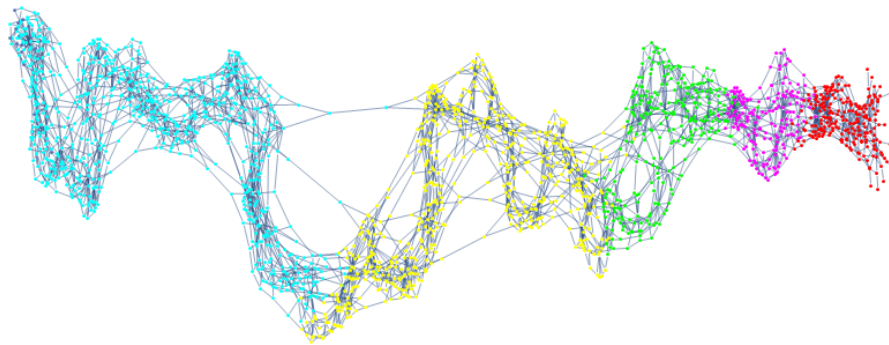


Fig. 3. Network visualization – Sphere function – Network growth phases

Table 1. Network statistics, Sphere function

Statistic	Value
Number of Vertices:	2010
Number of Edges:	3980
Shortest Path Length:	39
Particles on Shortest Path:	12
Mean Betweenness Centrality:	13781.8
Mean Degree Centrality:	3.86368
Mean Closeness Centrality:	0.0701754
Mean Clustering Coefficient:	0.197463
Graph Density:	0.00192319

4.2 Rosenbrock function

The Rosenbrock function represents smooth fitness landscape. In higher dimension this function is multimodal. Similarly, to the previous sub-section, Figure 4 presents the visualization of the network. the shortest path is highlighted. The nodes and links belonging to different particles are distinguished by different colors in Figure 5. The growth of the networks is presented in Figure 6. The network statistics are presented in Table 2.

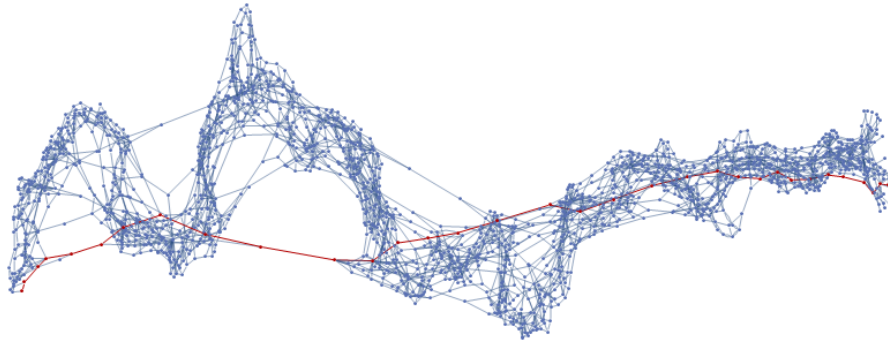


Fig. 4. Network visualization – Rosenbrock function – Shortest path highlighted

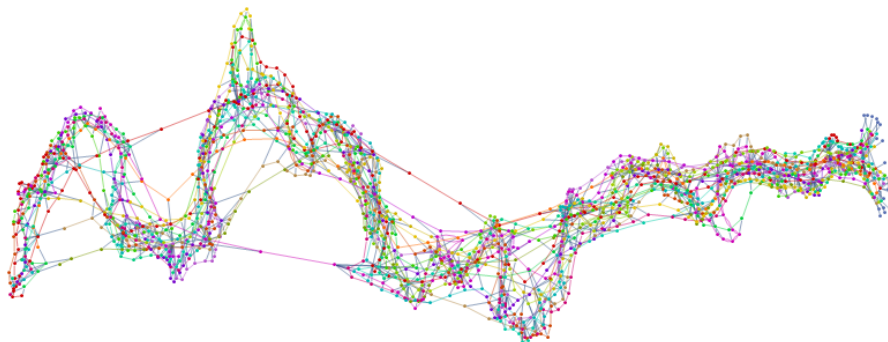


Fig. 5. Network visualization – Rosenbrock function – Particles paths in network

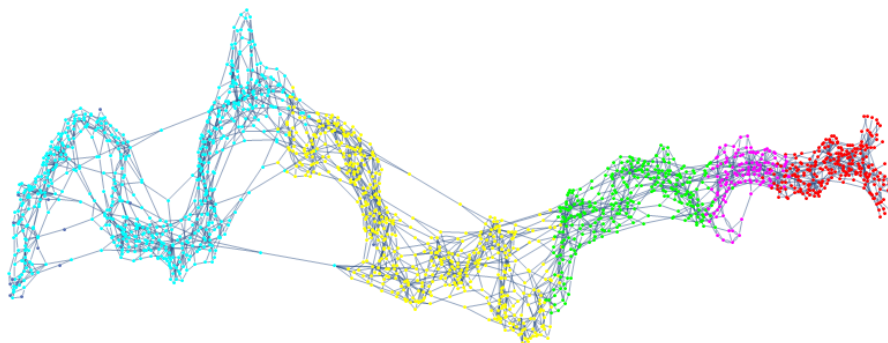


Fig. 6. Network visualization – Rosenbrock function – Network growth phases

Table 2. Network statistics, Rosenbrock function

Statistic	Value
Number of Vertices:	1793
Number of Edges:	3546
Shortest Path Length:	33
Particles on Shortest Path:	11
Mean Betweenness Centrality:	10989.4
Mean Degree Centrality:	3.83714
Mean Closeness Centrality:	0.0776457
Mean Clustering Coefficient:	0.175279
Graph Density:	0.00214126

4.3 Rastrigin function

The Rastrigin function represents highly rugged multimodal fitness landscape. Figure 7 presents the visualization of the network and the shortest path. The nodes and links belonging to different particles are distinguished by different colors in Figure 8. The growth of the networks is presented in Figure 9. The network statistics are presented in Table 3.

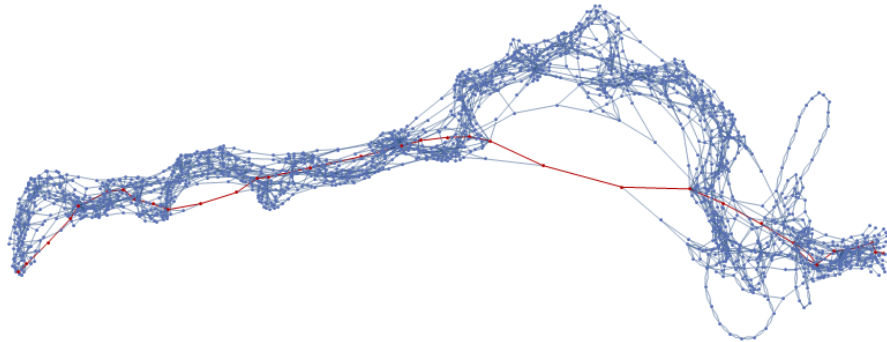
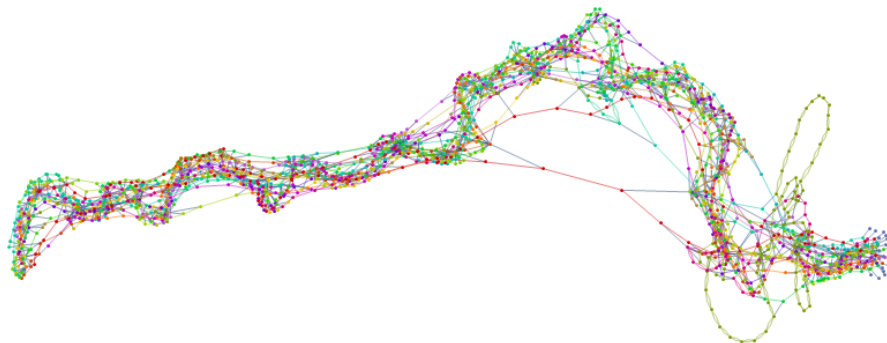
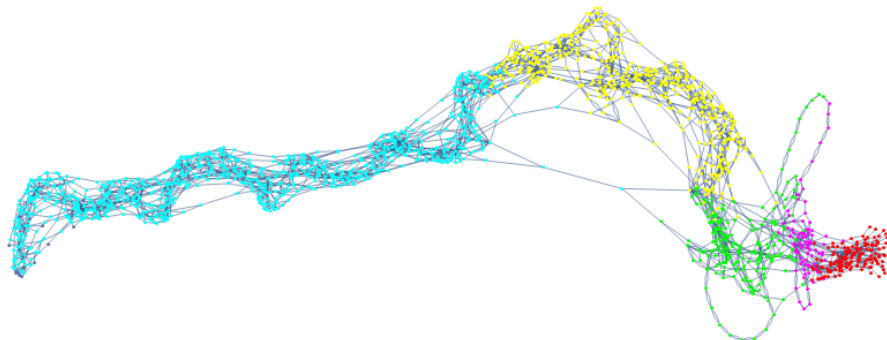
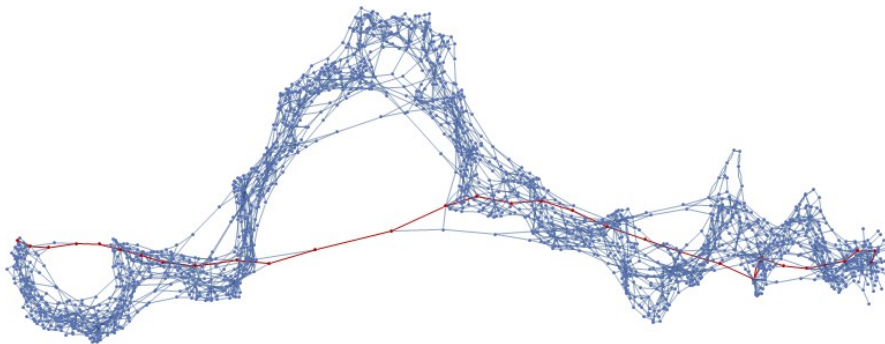
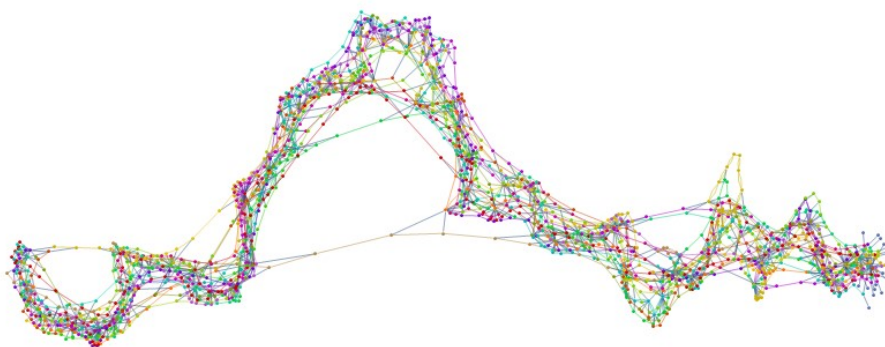
**Fig. 7.** Network visualization – Rastrigin function – Shortest path highlighted**Fig. 8.** Network visualization – Rastrigin function – Particles paths in network**Fig. 9.** Network visualization – Rastrigin function – Network growth phases

Table 3. Network statistics, Rastrigin function

Statistic	Value
Number of Vertices:	1839
Number of Edges:	3638
Shortest Path Length:	33
Particles on Shortest Path:	14
Mean Betweenness Centrality:	11190.6
Mean Degree Centrality:	3.78032
Mean Closeness Centrality:	0.0781403
Mean Clustering Coefficient:	0.173324
Graph Density:	0.00205675

4.4 Schwefel function

The Schwefel function represents mildly rugged multimodal fitness landscape. Figure 10 presents the visualization of the network; again, the shortest path is highlighted. The nodes and links belonging to different particles are distinguished by different colors in Figure 11. The growth of the networks is presented in Figure 12. The network statistics are presented in Table 4.

**Fig. 10.** Network visualization – Schwefel function – Shortest path highlighted**Fig. 11.** Network visualization – Schwefel function – Particles paths in network

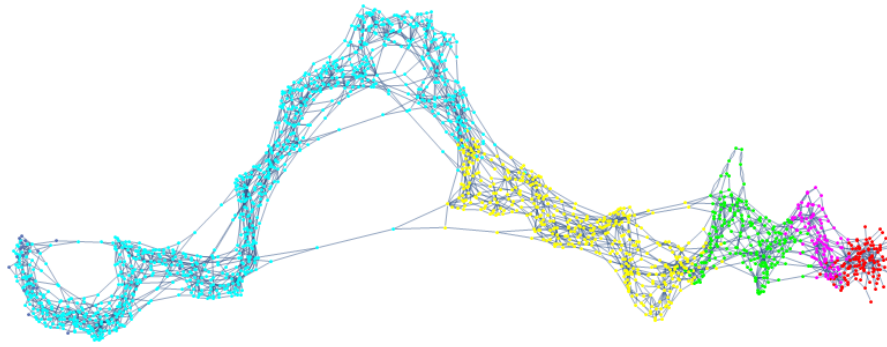


Fig. 12. Network visualization – Schwefel function – Network growth phases

Table 4. Network statistics, Schwefel function

Statistic	Value
Number of Vertices:	1796
Number of Edges:	3552
Shortest Path Length:	31
Particles on Shortest Path:	10
Mean Betweenness Centrality:	10341.3
Mean Degree Centrality:	3.85523
Mean Closeness Centrality:	0.0819255
Mean Clustering Coefficient:	0.198235
Graph Density:	0.00214776

5 Results discussion

In the previous section, several visualizations of constructed network structures were presented. Further, several statistics of the networks were presented. In this section, we analyze the above-presented results of the experiments.

The overall shapes and densities of the networks do not seem to be significantly different for different benchmark functions.

The number of edges and vertices in the network is highest for the simplest benchmark functions.

The shortest paths from first to the last node in the networks are surprisingly short and usually do not include all the particles from the swarm. We will focus in the future on the possibility of using this information for population decrease methods.

The centralities values seem to be very similar for all benchmark functions. Among the three indicators, the mean betweenness centrality seems to be the most promising indicator for future research.

According to Figures 3, 6, 9 and 12, the network growth is not proportional to the number of iterations of the algorithm. The majority of edges is created in the latest parts of the optimization process. This implies that the number of improvement in the swarm is highest in the final exploitation phase. However, the relative (or absolute) value of the improvement is not taken into consideration in the current method and this is one of the possible extensions for future research.

6 Conclusion

In this study, we investigated the characteristics of complex networks constructed from the inner dynamic of Particle Swarm Optimization algorithm. Several visualizations highlighting different aspects of the swarm dynamic were presented alongside with the network statistics. The main conclusions from the performed experiments are that the network statistics are mostly similar for different fitness landscapes and that the network growth is very nonproportional in time. Further, the shortest path in the network has been identified and will be investigated closely in the future.

We proposed several directions for future research. In future, we will focus on utilizing the network analysis into self-adaptive approaches for PSO that will allow the algorithm to dynamically adapt its parameters in order to improve its performance based on the information from complex network parameters and statistics.

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References

- [1] Kennedy J. and Eberhart R., “Particle swarm optimization,” in Proceedings of the IEEE International Conference on Neural Networks, 1995, pp. 1942–1948.
- [2] Shi Y. and Eberhart R., “A modified particle swarm optimizer,” in Proceedings of the IEEE International Conference on Evolutionary Computation (IEEE World Congress on Computational Intelligence), 1998, pp. 69–73.I. S.
- [3] Kennedy J., “The particle swarm: social adaptation of knowledge,” in Proceedings of the IEEE International Conference on Evolutionary Computation, 1997, pp. 303–308.”
- [4] Nickabadi A., Ebadzadeh M. M., Safabakhsh R., A novel particle swarm optimization algorithm with adaptive inertia weight, Applied Soft Computing, Volume 11, Issue 4, June 2011, Pages 3658-3670, ISSN 1568-4946.
- [5] van den Bergh, F., Engelbrecht, A.P.: A study of particle swarm optimization particle trajectories. Information Sciences 176(8), 937-971 (2006).
- [6] Cleghorn, C. W., Engelbrecht, A. P. (2015). Particle swarm variants: standardized convergence analysis. *Swarm Intelligence*, 9(2-3), 177-203.
- [7] Zelinka, I., Davendra, D., Senkerik, R., Jasek, R.: Do Evolutionary Algorithm Dynamics Create Complex Network Structures? *Complex Systems* 2, 0891–2513, 20, 127–140
- [8] Zelinka, I. Investigation on relationship between complex network and evolutionary algorithms dynamics, AIP Conference Proceedings 1389 (1) 1011–1014 (2011).
- [9] Zelinka, I., Davendra, D.D., Chadli, M., Senkerik, R., Dao, T.T., Skanderova, L.: Evolutionary Dynamics as The Structure of Complex Networks. In: Zelinka, I., Snasel, V., Abraham, A. (eds.) *Handbook of Optimization*. ISRL, vol. 38, pp. 215–243. Springer, Heidelberg (2013)
- [10] Davendra, D., Zelinka, I., Senkerik, R. and Pluhacek, M. Complex Network Analysis of Discrete Self-organising Migrating Algorithm, in: Zelinka, I. and Suganthan, P. and Chen, G. and Snasel, V. and Abraham, A. and Rossler, O. (Eds.) *Nostradamus 2014: Prediction, Modeling and Analysis of Complex Systems*, Advances in Intelligent Systems and Computing, Springer Berlin Heidelberg, pp. 161–174 (2014a).
- [11] Davendra, D., Zelinka, I., Metlicka, M., Senkerik, R., Pluhacek, M., "Complex network analysis of differential evolution algorithm applied to flowshop with no-wait problem," *Differential Evolution (SDE)*, 2014 IEEE Symposium on , vol., no., pp.1,8, 9-12 Dec. (2014b)
- [12] Digalakis, J. G., Margaritis, K. G. "On benchmarking functions for genetic algorithms." *International journal of computer mathematics* 77.4 (2001): 481-506.
- [13] Dieterich, J.M., Hartke B.. "Empirical review of standard benchmark functions using evolutionary global optimization." *arXiv preprint arXiv:1207.4318* (2012).
- [14] Wolfram Research, Inc., Mathematica, Version 11.1, Champaign, IL (2017).