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A Game Theoretic Competitive Supply Chain Network Model with Green Investments and Labour

Kurt Pace Debono✉**, Maria Kontorinaki, Monique Sciortino**
Department of Statistics and Operations Research, University of Malta, Malta
kurt.pace-debono.18@um.edu.mt✉, maria.kontorinaki@um.edu.mt, monique.sciortino@um.edu.mt

Abstract
In light of the recent severe Supply Chain (SC) disruptions that have occurred across multiple industries around the globe, three essential and linked themes have emerged in SC management: the well-being of employees, SC sustainability, and competition between SCs for limited resources. In this paper, we create a game-theoretic SC network model that incorporates together non-cooperative SC competition, employee productivity and engagement, and green investing. Each competing firm within the network seeks to maximise its profit by determining an optimal flow of products and allocation of green investments across the SC according to a predetermined budget. A carbon tax on emissions and consumer sustainability preferences are also included in the model. The model is solved using a Variational Inequality reformulation. The illustrative numerical examples presented in this paper have been inspired by the Maltese dairy industry and demonstrate the applicability of the model to real-world problems. The results highlight the significance of the employee engagement factor in enabling firms to adopt and realise more sustainable SC practices.

Keywords: *Green Supply Chain Management, Supply Chain Network Modelling, Optimisation, Game Theory, Employee Engagement, Labour, Sustainability*

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1 Introduction

The interconnectedness within and between Supply Chains (SCs), both on a local and global scale, has never been more apparent; SC disruptions caused by the global pandemic have occurred in multiple industries worldwide, while competition between SCs for limited natural and human resources has intensified.

With heightened awareness around climate change and its direct effects being felt across the globe, governments and international institutions are pushing to legislate towards greener initiatives. For example, the European Union launched the European Green Deal in 2019, which aims for the bloc to be climate neutral by 2050, with significant investments being planned to decarbonise significant polluters [6].

What is more, employers and researchers are realising that even employees are expecting more when it comes to the Environmental, Social and Governance (ESG) credentials of the company they work with. However, in Supply Chain Management (SCM) research, the human element is often overlooked. This was highlighted in [29], where over a hundred SCM researchers were asked which research themes they felt had been under-researched. Indeed, it was found that the most common answer was the people dimension of SCM, noting that only a few studies researched the "dynamics of consumers, managers, or other individual actors within a supply chain system". Nonetheless, we can still look at business management research as well as current trends that explore the link between employees and the environment. For example, in a 2020 global survey on employee expectations carried out amongst 14 million respondents, a 52% increase in environmental concern was registered over previous years [23]. This increase was significantly higher amongst the youngest generation of workers, with Generation Z respondents (born 1997-2007) registering a 128% increase, indicating that this issue is only going to become more important. This sentiment was amplified in a 2022 survey, which found that employees of purpose-driven employers are three times more likely to continue working with them, while 75% of respondents stated that they would be more likely to buy from a business that incorporates ESG credentials [24].

Various studies have theorised and tested the impact that going green has on employee productivity and *employee engagement*, the latter of which referring to the "level of commitment and involvement an employee has towards their organisation and its values" [3]. Employee engagement literature has honed in on the idea that employees who feel that the values of a firm align with their personal values, and that their contributions are meaningful, are more engaged in their work and, as a result, more productive [28]. This idea was incorporated into a theoretical model in [11] that links the impact of sustainability with employee engagement, with the main bridge between the two being the increased sense of meaningfulness that sustainable practices promote. Also, in [4], the authors sought to test the hypothesis that the adoption of sustainable practices is associated with increased labour productivity. Through an employee survey from 5220 firms, they found that firms that implemented environmental standards had one standard deviation of labour productivity higher than those that did not. In addition, the authors of [22], who studied the relationship between green human resource management and Green SCM (GSCM), found evidence to support the hypothesis that employees' empowerment in sustainable progression positively influences the implementation of GSCM practices.

GSCM concepts have entered the Supply Chain Network (SCN) modelling literature, with researchers adding environmental aspects to their models and studying their interactions with other parts of the model. For example, in [30], the authors modelled different carbon tax policies and found that such taxes can encourage firms to reduce emissions. The relationship between consumer environmental awareness and green SCs was studied in a model by [14], which found that the more aware consumers are, the more profitable sustainable firms will be. In [31], the authors developed a multi-period model that incorporated the relation between green investments and consumer purchasing behaviour, which highlighted that consumers have the power to encourage firms to go green. For an overview of the different components studied within green SC models, one can refer to [1].

The inclusion of labour in SCN models is a more recent area in SCN modelling literature, and to our knowledge is one that has not been studied in conjunction with green SCN models. It was first studied in [18], where the product flows in a competitive SCN were modelled as a function of labour, with firms also competing on the availability of human resources in the labour market. Following this initial paper which highlighted the importance of safeguarding employee health, the impact that investments in labour productivity can have on the SC profitability of a single firm was studied in [17, 19]. In [19], a single-firm model was created with the aim of optimising the firm's product flows and investments in labour productivity enhancements such as physical workplace improvements, training or health and safety, with labour availability being dependent on the wage offered by the firm. This model was extended to a multi-period model in [17], with labour productivity investments being incorporated into the demand-price function to model consumer sensitivity to the working conditions of the firm's workers. These papers both concluded that investments in labour productivity increased profits.

In light of this discussion, we can see that there is an interesting rationale behind modelling the interplay between green investments, employee engagement and labour productivity in our SCM model, which would be an original contribution to SCM literature. In this paper, we have developed a game-theoretic SC competition model, with a particular focus on the aspects of employee productivity and engagement, investments in green initiatives, and the link between the two. The model consists of a number of firms competing in an oligopolistic industry, whereby each firm seeks to maximise its profit by determining product flows and green investment allocations, within a predetermined budget, throughout the SCN. The element of labour is incorporated into the model by linking product flows with the amount of labour hours available to each firm and the employees' productivity. In turn, employee productivity is partially dependent on the employee engagement with the green investments that the firm makes. Furthermore, a carbon tax is included in the model, where each firm is taxed based on the amount of CO_2 emissions it produces. Since the firms compete within the same demand markets, the production and investment decisions made by each firm impact the profitability of all the firms. Assuming that each firm makes the decisions once, and at the same time as all the competing firms, the proposed model is created within a Game Theory (GT) framework as a static, non-cooperative game. Therefore, under this framework, we seek to find a Nash Equilibrium (NE) solution that ensures that no firm will be able to individually improve its profit, given the decisions of the other firms. In order to find such a NE, we use Variational Inequality (VI) theory to reformulate and solve the model.

To this end, in *Section 2*, we construct the SCN competition model with the inclusion of green investments and labour. This will be followed by a VI reformulation of the model and related VI theoretical results concerning the NE solution and its existence. In *Section 3*, we apply the model using scenarios inspired by the Maltese dairy industry and discuss the resultant managerial insights. Sensitivity analysis is also carried out to study the interplay between green investments and employee engagement introduced for the first time in our model, and its effect on SC profitability, demands and prices. Finally, in *Section 4*, we present a summary of the results and discuss the conclusions of this paper, as well as the direction for future research.

2 The SCN Competition Model with Green Investments and Labour

We consider an industry/network in which there are I *firms* seeking to maximise their profits by determining optimal product path flows and green investments. In this network, we assume that the firms compete non-cooperatively in the *delivery* of a substitutable product to customers in R *demand markets*. By delivery, we mean the entire set of processes carried out in order to convert raw materials into a product to be sold to customers, while note that a demand market could refer to an individual consumer, a business, an organisation, a retailer or a specific segment of customers.

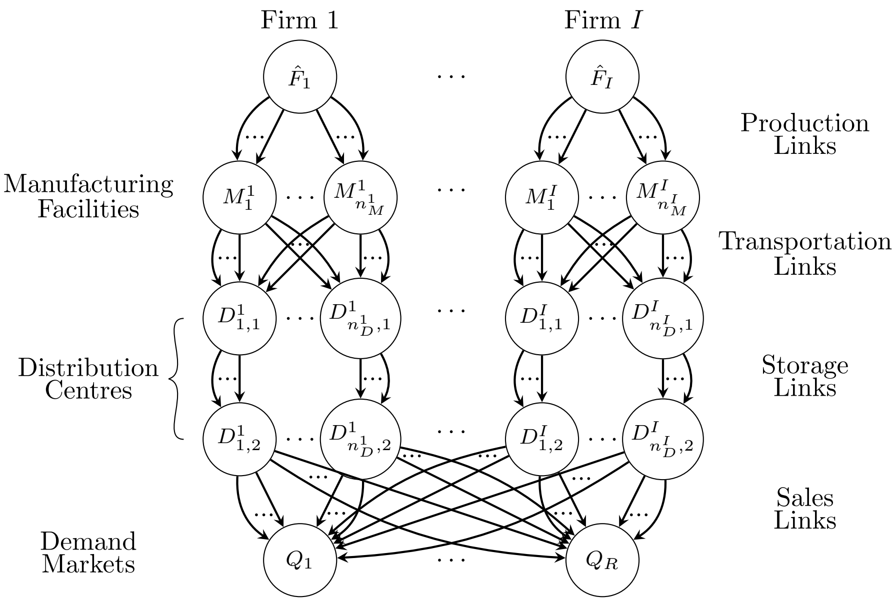

Figure 1: The SCN competition model topology.

2.1 Variables, Parameters and Functions

We denote the set of all firms by $\mathcal{I} = \{1, 2, \ldots, I\}$ and the set of all demand markets by $\mathcal{R} = \{1, 2, \ldots, R\}$. We can represent each firm $i \in \mathcal{I}$ competing in this industry as a SCN of its economic activities consisting of five tiers: the firm node with label $\hat{F}_i, i \in \mathcal{I}$; the firm's n_M^i manufacturing facilities $\{M_1^i, M_2^i, \ldots, M_{n_M^i}^i\}$; the first level of the firm's n_D^i distribution centres $\{D_{1,1}^i, D_{2,1}^i, \ldots, D_{n_D^i,1}^i\}$, representing the receiving of products from the manufacturing facilities; the second level of the same n_D^i distribution centres $\{D_{1,2}^i, D_{2,2}^i, \ldots, D_{n_D^i,2}^i\}$, representing the storage facilities; and, the demand market nodes with labels $Q_r, r \in \mathcal{R}$.

Each *link* between a pair of nodes in different tiers represents a SC process. The links between: the firm node and the manufacturing facilities represent the *production* processes of each firm; the manufacturing facilities and the first level of the distribution centres represent the *transportation* of the finished products; the first and second levels within the same distribution centres represent the *storage* of the products; the second level of the distribution centres and the demand markets represent the *sales* of the products. It is possible to have the same pair of nodes be connected by more than one link, adding the flexibility to allow for different options for each process, such as different production methods or modes of transport. Links can be grouped together to form a *path* (having one link of each type), which is a series of links that starts from a firm node and ends at a market node. Paths can be grouped into three sets: P_r^i, the set of all paths that join firm $i \in \mathcal{I}$ with demand market $r \in \mathcal{R}$; P^i, the set of all paths that join firm $i \in \mathcal{I}$ with all the R demand markets; and, P, the set of all paths in the SCN.

We depict the network of all the firms' nodes and links in the graph $G = (N, L)$ in Fig. 1, where N is the set of all nodes and L is the set of all links. Note that the SCs of the individual firms share no links with one another, thus we can group all the links representing the SC processes of firm $i \in \mathcal{I}$ into the set L^i.

Each firm $i \in \mathcal{I}$ seeks to maximise its profit by optimising two strategic vectors of decision variables: the vector of *product path flows* $\mathbf{x}^i = \{x_p\}_{p \in P^i}$ and the vector of *green investments* $\mathbf{v}^i = \{v_l\}_{l \in L^i}$. A product path flow, which we denote by x_p, refers to the flow of products along the path $p \in P^i$. On the other hand, v_l represents the amount invested in green initiatives on link $l \in L$. For example, on production links, these could represent the investment in solar panels or the introduction of environmentally friendly materials. On the transportation links, green investments could represent new electric vehicles, while on the storage links these could include the purchasing of energy efficient refrigeration units or the upgrading of climate control systems.

Central to this model, inspired by [18], is that the product path flows will be determined by the availability of labour hours $h_l, l \in L^i$, the firm $i \in \mathcal{I}$ has at its disposal. Two variables related to x_p will aid us in the formulation and interpretation of the model: the link flows f_l, $l \in L$ and demands d_r^i, $i \in \mathcal{I}$, $r \in \mathcal{R}$. The link flow f_l represents the amount of flow along link $l \in L$ in the SCN, while the demand d_r^i is the total amount of products delivered by firm $i \in \mathcal{I}$ to demand market $r \in \mathcal{R}$. Demands can be grouped into two vectors: $\mathbf{d}^i = \{d_r^i\}_{r \in \mathcal{R}}$, the vector of demands of firm $i \in \mathcal{I}$ at all demand markets, and $\mathbf{d}_r = \{d_r^i\}_{i \in \mathcal{I}}$, the vector of demands of all firms at demand market $r \in \mathcal{R}$.

Having defined the decision variables, we now define the functions that will make up our model's objective function. Firstly, we define the function that will determine the price of the competing products being sold. To this end, we define the demand price function $\rho_r^i(\mathbf{d}_r, \mathbf{v})$ which calculates the unit price of a product of firm $i \in \mathcal{I}$ at demand market $r \in \mathcal{R}$ as:

$$\rho_r^i(\mathbf{d}_r, \mathbf{v}) = \pi^i - \sum_{j\in\mathcal{I}} \sigma_r^j d_r^j + \eta_r^i \sum_{l\in L^i} v_l - \sum_{\substack{j\in\mathcal{I}\\ j\neq i}} \eta_r^j \sum_{l\in L^j} v_l,$$
$$\forall i \in \mathcal{I},\quad \forall r \in \mathcal{R}. \tag{1}$$

This function is an adaptation of the inverse demand function used in economics to express price as a function of the quantity demanded. The parameter π^i represents all the factors affecting the price of the product of firm $i \in \mathcal{I}$ other than the total demand and green investment. The parameters σ_r^j represent the effect that the demand of each of the firms' product at demand market $r \in \mathcal{R}$ has on the price of the product of firm $i \in \mathcal{I}$ at that same demand market. The parameters η_r^j represent the effect of the total green investment of each firm on the price of the product of firm $i \in \mathcal{I}$ at demand market $r \in \mathcal{R}$. This signifies the idea that consumers may be willing to pay more to purchase products that are environmentally friendly, a concept which was explored in [14, 27, 31]. Since the firms' products are substitutable, we note that the demands and investments of one firm also impacts the prices of other competing firms' products.

Next, we define the function that will capture the production/transportation/storage costs associated with the link flow f_l, as well as additional costs associated with green investments. Note that labour costs will not be considered in this function and will be inserted into the objective function separately by multiplying the number of hours worked h_l with the hourly wage ω_l. We define the operational cost function associated with link $l \in L$ as:

$$\hat{c}_l(f_l, v_l) = \gamma_l f_l^2 + \mu_l f_l v_l, \quad \forall l \in L. \tag{2}$$

The parameter γ_l represents the cost per unit squared of f_l. The term f_l is squared to model the economic concept of marginal cost; as the flow along a link l nears its maximum capacity, which is dictated by the upper bound on labour available, denoted by $\bar{h}_l$, the cost per unit increases [27]. The parameter μ_l represents the additional marginal cost per unit of flow that may arise out of the investment, such as increased maintenance requirements.

Since one of the main features of our model is the inclusion of labour, we construct a function that relates the amount of labour hours h_l worked with the product output on each link $l \in L$. A novel feature of our model is the relation of the amount v_l invested in green initiatives with productivity. To this end, we define the labour productivity function associated with link $l \in L$ as:

$$\hat{g}_l(h_l, v_l) = (\alpha_l + \beta_l v_l) h_l, \quad \forall l \in L. \tag{3}$$

The parameter α_l represents the factor directly relating the labour input to production output, such that one labour hour on link $l \in L$ produces α_l units of flow. On the other hand, the parameter β_l represents the impact that green investments have on the productivity of employees. This can be interpreted as a metric of employee engagement, whereby the more engaged employees are with the firm's investments, the more productive they are at their jobs.

We also include a function that tracks the amount of carbon emissions being generated throughout the SCN. The emissions function associated with link $l \in L$ can be defined as:

$$\hat{e}_l(f_l, v_l) = \xi_l f_l - \varphi_l f_l v_l, \quad \forall l \in L. \tag{4}$$

In this equation, similar to that defined in [31], the CO_2 emissions in tonnes are calculated as a function of the product flows and green investments. A relationship is modelled between the product flows and the emissions on a link $l \in L$, with every unit flow creating ξ_l units of CO_2. However, for every €1 invested in green initiatives, the emissions generated by one unit of flow are reduced by φ_l.

Finally, we model the introduction of a carbon tax on emissions. The carbon tax function associated with link $l \in L$ can be defined as:

$$\hat{t}_l(\hat{e}_l(f_l, v_l)) = \tau \hat{e}_l(f_l, v_l), \quad \forall l \in L, \tag{5}$$

where τ is the flat tax rate per tonne of CO_2 emitted. For example, τ could be equal to €50 per tonne emitted.

2.2 Objective Function and Constraints

Recall that each firm $i \in \mathcal{I}$ seeks to maximise its profit by deciding its strategic product flows and green investments. Thus, we define the objective function of firm $i \in \mathcal{I}$ as the profit function:

$$\begin{aligned} U^i = &\sum_{r=1}^{R} \rho_r^i(\mathbf{d}_r, \mathbf{v}) d_r^i - \sum_{l\in L^i} \hat{c}_l(f_l, v_l) - \sum_{l\in L^i} \omega_l h_l \\ &- \sum_{l\in L^i} \hat{t}_l(\hat{e}_l(f_l, v_l)) - \sum_{l\in L^i} v_l. \end{aligned} \tag{6}$$

The first term $\sum_{r=1}^{R} \rho_r^i(\mathbf{d}_r, \mathbf{v}) d_r^i$ of (6) is the total revenue of firm $i \in \mathcal{I}$ across the R demand markets, calculated by multiplying the price of the product of firm $i \in \mathcal{I}$ at demand market $r \in \mathcal{R}$ by the demand of that product in that market. To arrive at the profit figure for firm $i \in \mathcal{I}$, we then subtract from the total revenue term the total operational costs $\sum_{l\in L^i} \hat{c}_l(f_l, v_l)$, wages $\sum_{l\in L^i} \omega_l h_l$, carbon taxes $\sum_{l\in L^i} \hat{t}_l(\hat{e}_l(f_l, v_l))$ and investments in green initiatives $\sum_{l\in L^i} v_l$ across the n_{L^i} SC process links of the firm.

The optimisation of (6) is subject to a number of constraints. First, we require that the flow f_l along a link $l \in L^i$ equals the sum of that product's flow along all the paths x_p that contain that link, such that:

$$f_l = \sum_{p\in P^i} x_p \delta_{l,p}, \quad \forall l \in L^i, \quad \forall i \in \mathcal{I}, \tag{7}$$

where $\delta_{l,p}$ is a parameter that indicates whether link l is contained in path p or not; $\delta_{l,p} = 1$ if link $l \in L$ is contained in path $p \in P$ and $\delta_{l,p} = 0$ otherwise. Also, the path flows must be non-negative:

$$x_p \geq 0, \quad \forall p \in P^i, \quad \forall i \in \mathcal{I}. \tag{8}$$

Additionally, we ensure that the demand for the product of firm $i \in \mathcal{I}$ at market $r \in \mathcal{R}$ is satisfied by the sum of product flows along all the paths $p \in P_r^i$ starting from said firm and ending at said demand market:

$$\sum_{p \in P_r^i} x_p = d_r^i, \quad \forall i \in \mathcal{I}, \quad \forall r \in \mathcal{R}. \tag{9}$$

To relate flows with labour, we equate the product flow f_l on link $l \in L^i, i \in \mathcal{I}$, to the labour productivity function (3):

$$f_l = (\alpha_l + \beta_l v_l) h_l, \quad \forall l \in L^i, \quad \forall i \in \mathcal{I}, \tag{10}$$

which is capped by the upper bound $\bar{h}_l$ that we set on the total labour hours h_l available on link $l \in L^i, i \in \mathcal{I}$:

$$h_l \leq \bar{h}_l, \quad \forall l \in L^i, \quad \forall i \in \mathcal{I}. \tag{11}$$

The last two sets of constraints which we consider relate to the green investments made by the firms: First, we set bounds for the green investment v_l, i.e.,:

$$v_l^{min} f_l \leq v_l \leq v_l^{max} f_l, \quad \forall l \in L^i, \quad \forall i \in \mathcal{I}, \tag{12}$$

where v_l^{min} and v_l^{max} denote the minimum and maximum amounts of green investment per unit flow through link $l \in L^i$, respectively. For most links, v_l^{min} can be defined as 0, meaning no minimum investment would be required. However, there could exist *capital-intensive* links with corresponding positive v_l^{min}, which cater for scenarios where firms deem a minimum investment amount per unit of flow necessary to set up such a link. For example, a transportation link could represent the option a firm has to invest in an electric vehicle; for such an investment to be feasible, the minimum investment required per unit of flow to use this link would be the cost of one vehicle divided by the amount of units projected to flow through the link. By multiplying the parameters v_l^{min} and v_l^{max} by the flow variable f_l in (12), we ensure that if a firm decides not to make use of a link, then no investment will be made, i.e., $f_l = 0 \implies v_l = 0$. Second, the total budget constraint for firm $i \in \mathcal{I}$, where the total sum of green investments over the firm's entire set of links L^i cannot exceed the firm's budget Θ^i, i.e.,:

$$\sum_{l \in L^i} v_l \leq \Theta^i, \quad \forall i \in \mathcal{I}. \tag{13}$$

Considering the above, the optimisation problem faced by each firm is therefore to maximise its profit (6) subject to constraints (7) - (13).

2.3 Variational Inequality Reformulation

To aid the reformulation of the objective function into a VI problem, we rewrite the optimisation problem of each firm $i \in \mathcal{I}$ in terms of the path flow variables $\mathbf{x} = \{\mathbf{x}^i\}_{i \in \mathcal{I}}$ and green investment variables $\mathbf{v} = \{\mathbf{v}^i\}_{i \in \mathcal{I}}$.

By constraint (7), we can replace f_l with $\sum_{p \in P} x_p \delta_{l,p}$ wherever it appears in the objective function (6), as well as in the green investment constraint (12), giving us the following constraint:

$$v_l^{min} \sum_{p \in P} x_p \delta_{l,p} \leq v_l \leq v_l^{max} \sum_{p \in P} x_p \delta_{l,p}. \tag{14}$$

A similar replacement can be done for the demand d_r^i using the relation in constraint (9). Constraints (7) and (10) can be equated to each other, and then can be further combined with constraint (11) to give us the following:

$$\sum_{p \in P} x_p \delta_{l,p} \leq (\alpha_l + \beta_l v_l) \bar{h}_l, \; \forall l \in L. \tag{15}$$

Thus, our model aims to maximise the profit of each firm $i \in \mathcal{I}$

$$\tilde{U}^i(\mathbf{x}, \mathbf{v}) = \sum_{r=1}^{R} \tilde{\rho}_r^i(\mathbf{x}, \mathbf{v}) \sum_{p \in P_r^i} x_p - \sum_{l \in L^i} \tilde{c}_l(\mathbf{x}, v_l) - \sum_{l \in L^i} \frac{\omega_l}{\alpha_l + \beta_l v_l} \sum_{p \in P} x_p \delta_{l,p} - \sum_{l \in L^i} \hat{t}_l(\tilde{e}_l(\mathbf{x}, v_l)) - \sum_{l \in L^i} v_l, \tag{16}$$

subject to constraints (8), (13), (14) and (15), where $\tilde{\rho}_r^i(\mathbf{x}, \mathbf{v}) = \rho_r^i(\mathbf{d}_r, \mathbf{v})$, $\tilde{c}_l(\mathbf{x}, v_l) = \hat{c}_l(f_l, v_l)$ and $\tilde{e}_l(\mathbf{x}, v_l) = \hat{e}_l(f_l, v_l)$ for every $i \in I$, $r \in R$ and $l \in L^i$. We can define the feasible set of this problem for each firm $i \in \mathcal{I}$ as:

$$K^i \equiv \{(\mathbf{x}^i, \mathbf{v}^i) \mid (8), (13), (14) \,\&\, (15) \text{ hold}\}. \tag{17}$$

However, looking at the objective function (16), we notice that the profit $\tilde{U}^i(\mathbf{x}, \mathbf{v})$ of firm $i \in \mathcal{I}$ is determined not only by the firm's optimal choice of $(\mathbf{x}^i, \mathbf{v}^i)$, but also by its competitors' decisions for their own product flows and green investments. Thus, we can use Game Theory (GT) to solve the SCN optimisation problem using the framework of the non-cooperative game $\langle \mathcal{I}, (\mathbf{x}, \mathbf{v}), \mathbf{U} \rangle$, where $\mathcal{I}$ is the set of firms; $(\mathbf{x}, \mathbf{v})$ is the tuple of strategies consisting of the product flow and green investment vectors $\mathbf{x}^i$ and $\mathbf{v}^i$ of each firm $i \in \mathcal{I}$; and, $\mathbf{U} = \{\tilde{U}^i(\mathbf{x}, \mathbf{v})\}_{i \in \mathcal{I}}$ is the set of objective functions of all firms. We define the feasible set of this oligopolistic competition problem as the set:

$$K = \prod_{i=1}^{I} K^i = K^1 \times K^2 \times \cdots \times K^I. \tag{18}$$

The optimal solution in such a GT framework would be what is known as a Nash Equilibrium (NE) solution [21]. To define the form of a NE solution, let us first define the *decision vectors of the competitors* of firm $i \in \mathcal{I}$, relating to the product flows and the green investments, i.e.,:

$$\mathbf{x}^{-i} = (\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^{i-1}, \mathbf{x}^{i+1}, \ldots, \mathbf{x}^I), \quad \forall i \in \mathcal{I},$$

$$\mathbf{v}^{-i} = (\mathbf{v}^1, \mathbf{v}^2, \ldots, \mathbf{v}^{i-1}, \mathbf{v}^{i+1}, \ldots, \mathbf{v}^I), \quad \forall i \in \mathcal{I},$$

respectively. For our model, a NE $(\mathbf{x}^*, \mathbf{v}^*)$ is established if a firm $i \in \mathcal{I}$ cannot individually improve its profit by changing its decisions $(\mathbf{x}^{i*}, \mathbf{v}^{i*})$, given the other firms' decisions $(\mathbf{x}^{-i*}, \mathbf{v}^{-i*})$.

Definition (Model Nash Equilibrium)**.** *A tuple of path flows and green investments* $(\mathbf{x}^*, \mathbf{v}^*)$ *is said to be a NE for the competitive SCN model if for each firm* $i \in \mathcal{I}$*:*

$$\tilde{U}^i(\mathbf{x}^{i*}, \mathbf{v}^{i*}, \mathbf{x}^{-i*}, \mathbf{v}^{-i*}) \geq \tilde{U}^i(\mathbf{x}^i, \mathbf{v}^i, \mathbf{x}^{-i*}, \mathbf{v}^{-i*}), \quad \forall (\mathbf{x}^i, \mathbf{v}^i) \in K^i. \tag{19}$$

Under this definition, an optimal solution to our model is not focused on maximising one individual firm's profit, but instead optimising all the firms' decision vectors *concurrently* in such a way that the SCN is in equilibrium.

2.4 Related VI Theoretical Results

VI theory provides the tools necessary to find the equilibrium of mathematical problems, a solution concept that is central to GT. Thus, by reformulating our GT competitive SCN model into a VI problem, we can proceed with finding a NE solution. A background on related theorems and algorithms can be found in [9, 16, 20]. For our model, we will apply the following theorem, the proof of which can be found in [10].

Theorem (NE solution)**.** *Assume that for each firm* $i \in \mathcal{I}$*, the profit function* $\tilde{U}^i(\mathbf{x}, \mathbf{v})$ *is continuously differentiable and concave in* $\mathbf{x}$ *and* $\mathbf{v}$*. Also, assume that the feasible set* K *is convex. Then,* $(\mathbf{x}^*, \mathbf{v}^*) \in K$ *is said to be a NE for our competitive SCN model if and only if it satisfies the VI:*

$$-\sum_{i=1}^{I} \left\langle \nabla_{\mathbf{x}_i} \tilde{U}^i(\mathbf{x}^*, \mathbf{v}^*), \mathbf{x}^i - \mathbf{x}^{i*} \right\rangle - \sum_{i=1}^{I} \left\langle \nabla_{\mathbf{v}_i} \tilde{U}^i(\mathbf{x}^*, \mathbf{v}^*), \mathbf{v}^i - \mathbf{v}^{i*} \right\rangle \geq 0, \quad \forall (\mathbf{x}, \mathbf{v}) \in K, \tag{20}$$

where $\langle \cdot, \cdot \rangle$ *is the inner product in the* n*-dimensional Euclidean space.*

The theorem above links the solution of a VI to a NE solution. We will now proceed with explaining how the assumptions of the theorem above hold for our SCN model. The feasible set K in (18) is convex as it is the Cartesian product of convex sets (each K^i, $i \in \mathcal{I}$, is constructed by considering simple and linear bounds for the decision variables). Also, from the form of (16), it can easily be observed that the utility functions are continuously differentiable in K^i, for each $i \in \mathcal{I}$. The concavity of the utility functions is typically assumed throughout the SCN modelling literature [18, 30, 31, 27].

For the sake of completeness, we also provide an existence result that is relevant to our model [26].

Theorem (Existence)**.** *The existence of a NE for our competitive SCN model is guaranteed under the compactness of the feasible set* K *in (18) and continuous differentiability of each* $\tilde{U}^i$*,* $i \in \mathcal{I}$*.*

Since $K \subset \mathbb{R}^I$, compactness of K follows from the compactness of K^i's. Each K^i is compact. i.e., closed and bounded. From (8), we have that $x_p \geq 0$, $\forall p \in P^i$, $\forall i \in \mathcal{I}$. Thus, each path flow $f_l = \sum_{p \in P} x_p \delta_{l,p}$ is bounded below. Moreover, from (15), we know that each path flow f_l is bounded above by $(\alpha_l + \beta_l v_l)\bar{h}_l$. From (14), we have that $v_l^{min} f_l \leq v_l \leq v_l^{max} f_l$, $\forall l \in L^i$, $\forall i \in \mathcal{I}$. Thus, v_l is bounded below and above. Inequality (13) is another constraint on the v_l's, such that $\sum_{l \in L^i} v_l \leq \Theta^i$, $\forall i \in \mathcal{I}$, imposing an upper bound on the sum of all green investments for each firm. Since all the inequalities that make up each K^i are not strict, we have that each K^i is closed as well.

To solve our VI problem, we will be making use of the *Extragradient Algorithm* [13]. Assuming that the function F is monotone and Lipschitz continuous, this algorithm is guaranteed to converge to a solution with a polynomial rate of convergence (see Theorem 12.6.4 in [9]).

Algorithm 1 Extragradient Algorithm

Step 0: ***Initialisation***
Set initial solution $\mathbf{z}^0 = (\mathbf{x}^0, \mathbf{v}^0) \in K$.
Let the iteration counter $t = 1$.
Let ζ be a scalar such that $0 < \zeta \leq \frac{1}{\mathcal{L}}$, where $\mathcal{L}$ is the Lipschitz continuity constant.
Set tolerance $\varepsilon > 0$.

Step 1: ***Computation***
Compute $\bar{\mathbf{z}}^{t-1}$ by solving the VI subproblem:
$\langle \bar{\mathbf{z}}^{t-1} + \zeta F(\mathbf{z}^{t-1}) - \mathbf{z}^{t-1}, \mathbf{z} - \bar{\mathbf{z}}^{t-1} \rangle \geq 0, \quad \forall \mathbf{z} \in K.$

Step 2: ***Adaptation***
Compute $\mathbf{z}^t$ by solving the VI subproblem:
$\langle \mathbf{z}^t + \zeta F(\bar{\mathbf{z}}^{t-1}) - \mathbf{z}^{t-1}, \mathbf{z} - \mathbf{z}^t \rangle \geq 0, \quad \forall \mathbf{z} \in K.$

Step 3: ***Convergence Verification***
If $|\mathbf{z}^t - \mathbf{z}^{t-1}| \leq \varepsilon$, then stop; else, set $t := t + 1$ and go to Step 1.

3 Numerical Application

In this section, we will be constructing scenarios inspired by the Maltese dairy industry to illustrate the properties of the proposed model, while sensitivity analysis on the most important parameters will also be carried out.

3.1 Data and Scenarios

According to [7], the average cost of raw milk in Malta in 2021 was 57.44 cents per kilogram (kg). Competition in this industry exists between farms, as well as with alternative milk products such as dairy free and long-life milk. A breakdown of milk production costs was studied in [8]; on average, labour costs represented around 23% of these costs in 2019, with feed, machinery and equipment costs representing a further 39%, and general operating costs representing the remaining 38%. Interestingly, the authors of the latter report highlight that better recognition of labour costs is required, with the amount of labour hours, experience and knowledge not corresponding with the average labour costs computed. The median annual salary for persons working with livestock in 2018 was €12,500 per annum [12], equating to a wage of approximately €6 per hour. Estimating productivity, in 2019 the average yield was 6,843kg of milk per cow, with the average amount of cows per farm being 67.9 [5], meaning that each Maltese farm had an average output of 464,639kg per annum. With an average labour input of 6,115 hours per farm per annum [5], the milk production per hour of labour can therefore be estimated as approximately 76kg/hour. With regards to emissions, in a study of twelve Maltese dairy farms [25], it was found that the amount of CO_2 equivalent emissions per kilogram of milk ranged between 1.14kg and 3.00kg. In an analysis of the entire local dairy production process, the author of [2] commented that almost half of the energy consumption occurs during the refrigeration (storage) stage.

Based on this data, let us consider a scenario where two competing dairy farms, Farm 1 and Farm 2, would like to optimise the flow of milk and green investments within their SCs, depicted in the SCN topolgy in Fig. 2. Farm 1 has two production facilities M_1^1 and M_2^1 and a distribution centre D_1^1, while Farm 2 has one production facility M_1^2 and one distribution centre D_1^2. Both farms serve two demand markets, Q_1 and Q_2.

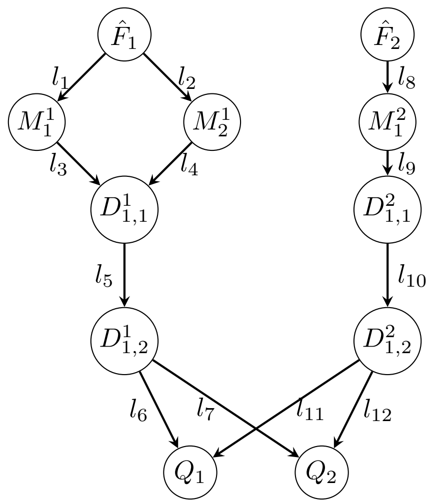

Figure 2: SCN topology for two competing farms.

We define the operational cost (in cents), labour productivity (in kgs of milk produced per labour hour) and carbon emissions (in kgs of CO_2 per kg of flow of milk) functions in Table 1. Looking at the parameters in this table, we note that the cost and emissions parameters for Farm 1's second production facility M_2^1 are lower since it is equipped with more modern machinery. Similarly, Farm 2 has recently invested heavily in the latest technologies across its SC, thus we notice lower parameters in the emissions functions for Farm 2 compared to those of Farm 1, as well as some slightly higher costs due to using more sustainable materials.

With regards to the cost of labour, Farm 1 pays its workers €6 per hour, while Farm 2 opts to pay a higher wage of €7 per hour. Thus, $\omega_l = 600,\ \forall l \in L^1$ and $\omega_l = 700,\ \forall l \in L^2$. Assuming a 40 hour week and that Farm 1 employs 3 full-time workers while Farm 2 employs 6 full-time workers and one part timer, the upper bounds on labour $\bar{h}_l = 120,\ \forall l \in L^1$ and $\bar{h}_l = 250,\ \forall l \in L^2$.

In the demand price functions of Farm 1 (in cents), we set the baseline price at $\pi^1 = 100$, which falls within the range of current market prices. When setting the parameters η, we specify higher values at Market 2 to simulate customers at this market being more environmentally conscious than those at Market 1. Thus, we set $\sigma_1^1 = \sigma_2^1 = 0.001$, $\sigma_1^2 = \sigma_2^2 = 0.0003$, $\eta_1^1 = 0.001$, $\eta_2^1 = 0.0015$, $\eta_1^2 = 0.0008$ and $\eta_2^2 = 0.001$. For Farm 2's functions, we set $\pi^2 = 130$, $\sigma_1^1 = \sigma_2^1 = 0.0002$, $\sigma_1^2 = \sigma_2^2 = 0.001$, $\eta_1^1 = 0.0008$, $\eta_2^1 = 0.001$, $\eta_1^2 = 0.001$ and $\eta_2^2 = 0.0015$. We note that Farm 2 has a higher baseline price of 130 cents, owing to the significant investments it has made to set up sustainable operations.

To combat a newly introduced carbon tax, management at the farms would like to invest in green initiatives in order to reduce their CO_2 emissions. To this end, they allocate a budget of €5,000 and set $v_l^{min} = 0, v_l^{max} = 0.5\ \forall l \in L$, such that they would not like to spend more than 0.5c per kg of flow of milk on any specific link.

We can define the paths in this model as $p_1{=}(l_1, l_3, l_5, l_6)$, $p_2{=}(l_2, l_4, l_5, l_6)$, $p_3{=}(l_1, l_3, l_5, l_7)$, $p_4 = (l_2, l_4, l_5, l_7)$, $p_5 = (l_8, l_9, l_{10}, l_{11})$, and $p_6 = (l_8, l_9, l_{10}, l_{12})$. The set of links $L = \{l_1, l_2, \ldots, l_{12}\}$ can be split into those in Farm 1's SC as $L^1 = \{l_1, l_2, \ldots, l_7\}$ and those in Farm 2's as $L^2 = \{l_8, l_9, \ldots, l_{12}\}$.

Scenario 1 (Base Case)**.** We solve the base scenario in MATLAB[1] using the Extragradient Algorithm with a step size of $\zeta = 50$, tolerance of $\varepsilon = 0.01$ and all path flows x_p initialised at 3000 and green investments v_l initialised at 0. The parameter ζ has been chosen by performing a grid search and by assuming a sufficiently large Lipschitz constant $\mathcal{L}$. We obtain the following NE solution:

$$x_{p_1}^* = 4599,\ x_{p_2}^* = 5780,\ x_{p_3}^* = 4900,\ x_{p_4}^* = 6080,$$
$$x_{p_5}^* = 16543,\ x_{p_6}^* = 17207,$$

with a profit of €11,804 for Farm 1 and €24,603 for Farm 2, equilibrium demands $d_1^{1*} = 10379$ and $d_2^{1*} = 10981$ for Farm 1 with corresponding prices $\rho_1^1(\mathbf{d}_1^*, \mathbf{v}^*) = 86$ and $\rho_2^1(\mathbf{d}_2^*, \mathbf{v}^*) = 86$ c/kg, and equilibrium demands $d_1^{2*} = 16543$ and $d_2^{2*} = 17207$ for

[1] The code for the numerical scenarios can be accessed at: https://github.com/kurtpacedebono/A-Game-Theoretic-Competitive-Supply-Chain-Network-Model-with-Green-Investments-and-Labour.git

Table 1: Operational cost, labour productivity and carbon emissions functions for Farms 1 and 2.

Link	$\hat{c}_l(f_l, v_l)$	$\hat{g}_l(h_l, v_l)$	$\hat{e}_l(f_l, v_l)$
Farm 1:			
l_1	$0.00042f_{l_1}^2 + 0.0005f_{l_1}v_{l_1}$	$(76 + 0.01v_{l_1})h_{l_1}$	$0.45f_{l_1} - 0.000005f_{l_1}v_{l_1}$
l_2	$0.00032f_{l_2}^2 + 0.0004f_{l_2}v_{l_2}$	$(80 + 0.01v_{l_2})h_{l_2}$	$0.34f_{l_2} - 0.000004f_{l_2}v_{l_2}$
l_3	$0.00008f_{l_3}^2 + 0.0001f_{l_3}v_{l_3}$	$(300 + 0.01v_{l_3})h_{l_3}$	$0.05f_{l_3} - 0.000001f_{l_3}v_{l_3}$
l_4	$0.00008f_{l_4}^2 + 0.0001f_{l_4}v_{l_4}$	$(300 + 0.01v_{l_4})h_{l_4}$	$0.05f_{l_4} - 0.000001f_{l_4}v_{l_4}$
l_5	$0.0001f_{l_5}^2 + 0.0005f_{l_5}v_{l_5}$	$(150 + 0.01v_{l_5})h_{l_5}$	$0.45f_{l_5} - 0.000005f_{l_5}v_{l_5}$
l_6	$0.00008f_{l_6}^2 + 0.0001f_{l_6}v_{l_6}$	$(300 + 0.01v_{l_6})h_{l_6}$	$0.05f_{l_6} - 0.000001f_{l_6}v_{l_6}$
l_7	$0.00008f_{l_7}^2 + 0.0001f_{l_7}v_{l_7}$	$(300 + 0.01v_{l_7})h_{l_7}$	$0.05f_{l_7} - 0.000001f_{l_7}v_{l_7}$
Farm 2:			
l_8	$0.00052f_{l_8}^2 + 0.0001f_{l_8}v_{l_8}$	$(85 + 0.01v_{l_8})h_{l_8}$	$0.25f_{l_8} - 0.000003f_{l_8}v_{l_8}$
l_9	$0.0001f_{l_9}^2 + 0.0001f_{l_9}v_{l_9}$	$(450 + 0.01v_{l_9})h_{l_9}$	$0.01f_{l_9} - 0.000001f_{l_9}v_{l_9}$
l_{10}	$0.00006f_{l_{10}}^2 + 0.0001f_{l_{10}}v_{l_{10}}$	$(250 + 0.01v_{l_{10}})h_{l_{10}}$	$0.3f_{l_{10}} - 0.000004f_{l_{10}}v_{l_{10}}$
l_{11}	$0.00004f_{l_{11}}^2 + 0.0001f_{l_{11}}v_{l_{11}}$	$(450 + 0.01v_{l_{11}})h_{l_{11}}$	0
l_{12}	$0.00004f_{l_{12}}^2 + 0.0001f_{l_{12}}v_{l_{12}}$	$(450 + 0.01v_{l_{12}})h_{l_{12}}$	0

Farm 2 with corresponding prices $\rho_1^2(\mathbf{d}_1^*, \mathbf{v}^*) = 112$ and $\rho_2^2(\mathbf{d}_2^*, \mathbf{v}^*) = 113$ c/kg. The equilibrium link flows, green investments, labour requirements and emissions can be seen in Table 2.

Table 2: Results for Scenario 1.

Link	f_l^*	v_l^*	h_l^*	$\hat{e}_l(f_l^*, v_l^*)$
l_1	9500	316	120	4260
l_2	11860	1884	120	3943
l_3	9500	0	32	475
l_4	11860	0	40	593
l_5	21360	2800	120	9313
l_6	10379	0	35	519
l_7	10981	0	37	549
l_8	33750	5000	250	7931
l_9	33750	0	75	337
l_{10}	33750	0	135	10125
l_{11}	16543	0	37	0
l_{12}	17207	0	38	0

From these results, we can see how Farm 2 performs strongly in both markets, managing to attract higher demands whilst still maintaining higher prices that reflect the higher sustainability of the farm's practices. As a testament to this better environmental track record, Farm 2 produces 6.4% less total emissions than Farm 1 whilst having a 58% higher total flow of milk throughout its SC. Should Farm 1 wish to improve its position in the market, it should consider investing more into sustainable operations. These observations highlight the impact that competition can have in the market, such that if one firm in a SCN opts to go green and this is well received by the consumers, then this may have a domino effect and convince other competitors to become more sustainable themselves.

Scenario 2 (No Employee Engagement with Sustainability)**.** In this scenario, we explore what happens when we remove the increase in labour productivity experienced due to employee engagement with green investments. Thus, we set the parameter $\beta_l = 0$ for all links $l \in L$, affecting the productivity functions $\hat{g}_l(h_l, v_l)$, as well as the product flow constraints (15).

Solving this scenario, we obtain the following NE solution:

$$x_{p_1}^* = 4048,\ x_{p_2}^* = 4648,\ x_{p_3}^* = 4352,\ x_{p_4}^* = 4952,$$
$$x_{p_5}^* = 10294,\ x_{p_6}^* = 10956,$$

with a profit of €10,825 for Farm 1 and €18,462 for Farm 2, equilibrium demands $d_1^{1*} = 8695$ and $d_2^{1*} = 9305$ for Farm 1 with corresponding prices $\rho_1^1(\mathbf{d}_1^*, \mathbf{v}^*) = 89$ and $\rho_2^1(\mathbf{d}_2^*, \mathbf{v}^*) = 90$ c/kg, and equilibrium demands $d_1^{2*} = 10294$ and $d_2^{2*} = 10956$ for Farm 2 with corresponding prices $\rho_1^2(\mathbf{d}_1^*, \mathbf{v}^*) = 119$ and $\rho_2^2(\mathbf{d}_2^*, \mathbf{v}^*) = 120$ c/kg. We can note that the profit declined for both farms, declining by 8.3% for Farm 1 when compared to the Base Case, and by 25% for Farm 2. The dramatic decline for Farm 2 can be attributed to the significantly lower output from its manufacturing link; with productivity falling from 33,750kg to 21,250kg. We also note that the decrease in output, especially that experienced by Farm 2, has driven up the prices, with the two farms' prices increasing between 3.5% and 6.3% at the two demand markets. In Table 3, we note that due to the removal of the relationship between productivity and investments, we can see a shift in the way the €5,000 is invested by both firms. The results of this scenario therefore highlight the importance that employee engagement has on the effectiveness of green investments and the overall profitability of a SCN.

Scenario 3 (Labour Shortages)**.** Inspired by the COVID-19 pandemic, we construct a scenario where an outbreak occurs at Farm 1's manufacturing facility M_2^1 with corresponding link l_2, leaving only 10 labour hours available out of the usual 120. Solving such a scenario yields the following NE solution:

$$x_{p_1}^* = 7409,\ x_{p_2}^* = 249,\ x_{p_3}^* = 7710,\ x_{p_4}^* = 551,$$
$$x_{p_5}^* = 16543,\ x_{p_6}^* = 17207,$$

Table 3: Results for Scenario 2.

Link	f_l^*	v_l^*	h_l^*	$\hat{e}_l(f_l^*, v_l^*)$
l_1	8400	0	111	3780
l_2	9600	0	120	3264
l_3	8400	348	28	417
l_4	9600	0	32	480
l_5	18000	0	120	8100
l_6	8695	0	29	435
l_7	9305	4652	31	422
l_8	21250	0	250	5312
l_9	21250	0	47	212
l_{10}	21250	5000	85	5950
l_{11}	10294	0	23	0
l_{12}	10956	0	24	0

with a profit of €9,014 for Farm 1 and €25,378 for Farm 2, equilibrium demands $d_1^{1*} = 7659$ and $d_2^{1*} = 8261$ for Farm 1 with corresponding prices $\rho_1^1(\mathbf{d}_1^*, \mathbf{v}^*) = 88$ and $\rho_2^1(\mathbf{d}_2^*, \mathbf{v}^*) = 90$ c/kg, and equilibrium demands $d_1^{2*} = 16543$ and $d_2^{2*} = 17207$ for Farm 2 with corresponding prices $\rho_1^2(\mathbf{d}_1^*, \mathbf{v}^*) = 113$ and $\rho_2^2(\mathbf{d}_2^*, \mathbf{v}^*) = 114$ c/kg.

We can see that these results represent a 23.6% decrease in profit for Farm 1 when compared to the base case, owing to a 25.5% decrease in output capacity. From the equilibrium results in Table 4, we can see that even though production is ramped up at the first manufacturing facility M_1^1 to make up for the severely restricted capacity at M_2^1, this is not enough to make up for the lost output. We can also note how the €5,000 budget is fully allocated to the first manufacturing facility to increase the flow capacity as much as possible. This highlights the importance of safeguarding employee health, a theme that has emerged and been strongly prioritised throughout the pandemic.

Table 4: Results for Scenario 3.

Link	f_l^*	v_l^*	h_l^*	$\hat{e}_l(f_l^*, v_l^*)$
l_1	15120	5000	120	6426
l_2	800	0	10	272
l_3	15120	0	50	756
l_4	800	0	3	40
l_5	15920	0	106	7164
l_6	7659	0	26	383
l_7	8261	0	28	413
l_8	33750	5000	250	7931
l_9	33750	0	75	337
l_{10}	33750	0	135	10125
l_{11}	16543	0	37	0
l_{12}	17207	0	38	0

3.2 Sensitivity Analysis

Since the scenarios highlighted an important link between employees, green investments and profitability, using the same functions and topology, we will be conducting sensitivity analysis on the employee engagement with green investments β, and the green investment budget Θ.

To study the impact that employee engagement with green investments has on the SC of a firm, we consider productivity factors β between 0 and 1 kg/€ invested, increasing in increments of 0.05. We note that we vary the productivity factor for Farm 1, while those of Farm 2 remain fixed at 0.1kg/€. In Fig. 3, we can note that Farm 2's profits are not impacted by the variations in the employee engagement at Farm 1, which is to be expected. For Farm 1, up to a productivity factor of 0.25kg/€ invested, its profit increases at an average rate of 3.3% per 0.05 increment. However, at the 0.3kg/€ point we note that this profit takes a hit, initially decreasing by 14.4% and then climbing at an average rate of 0.5% thereafter. This drop can be attributed to the employees' perceptions and expectations regarding green investments. Keeping in mind that the budget Θ is fixed at €5,000, initially the ratio between the productivity factor and the budget results in an increase in profits due to the increased employee engagement. However, beyond the 0.25kg/€ point, productivity diminishes due to the gap between the employees' environmental expectations and what is actually being carried out by the firm. This is in line with findings that employees at purpose-driven firms are more likely to remain working with them [24], with employee engagement being one of the linking factors. This also highlights that the more environmentally aware employees are, the more they will demand from their employer. To confirm this thinking, we carried out the same sensitivity analysis again with a higher budget of €7,000 and could note that the inflection point occurred at a higher productivity factor of 0.35kg/€.

Finally, we carry out sensitivity analysis on the budget parameter Θ to study the impact the amount that a firm invests in green initiatives has on its SC. To this end, we consider budgets between €0 and €100,000 in increments of €5,000 for Farm 1, while keeping Farm 2's budget fixed at €10,000. From Fig. 4, we can see that an increase in budget leads to an increase in productivity, demands and ultimately profits for Farm 1. This in turn impacts Farm 2's performance, which loses market share and as a result sees a decline in profit. On average, profits for Farm 1 increase by €2,598 for every additional €5,000 invested, representing an average return on investment of 52%, while profits for Farm 2 decrease by €1,420 with every increment. Farm 1 overtakes Farm 2 in terms of profits at an investment level of €30,000, at which point Farm 1 invests 3 times that of Farm 2. At a budget of €90,000, Farm 1's profit reaches a maximum of €60,223, which cannot be improved beyond this point, regardless of the budget increase. This is because at this point, the ratio of green investments to link flows is at 0.5 for all links, which we recall is the upper bound per kg of flow. We can also see how Farm 1 prioritises Market 2 over Market

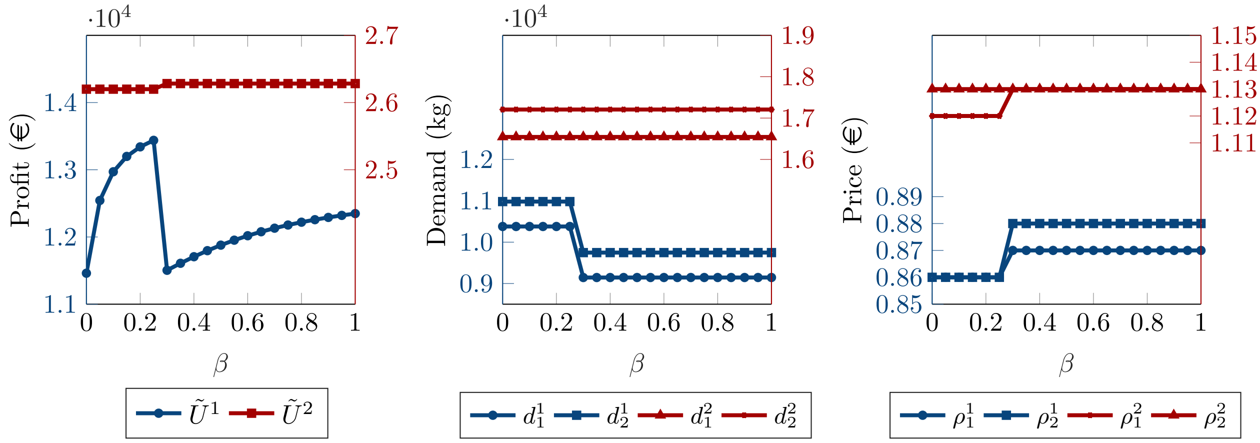

Figure 3: Equilibrium profits, demands and prices for different values of β.

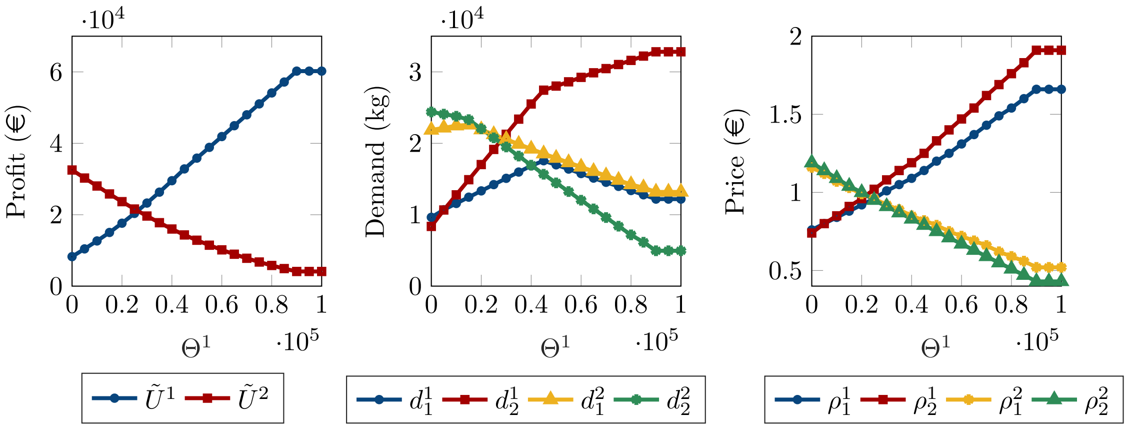

Figure 4: Equilibrium profits, demands and prices for different values of Θ.

1 due to the higher consumer sensitivity to green investments at that market ($\eta_2^1 > \eta_1^1$). Until a budget of €45,000, demand at Market 1 and Market 2 for Farm 1's product increases at an average rate of 6.9% and 14.3% per €5,000 budget increase, respectively. Beyond this point, the demand at Market 1 shrinks at an average rate of 4% while the demand at Market 2 continues to increase at an average rate of 2%, thus seeing how the demand at Market 2 cannibalises that at Market 1. The demand for Farm 2's product is also impacted, with this decreasing by an average 2.8% and 8.4% at the two markets, respectively. The increases in investments are passed on to the consumers through an increase in price, with prices at Market 1 and Market 2 increasing by 4.4% and 5.4% respectively for Farm 1, while decreasing by the same percentages for Farm 2.

4 Conclusions

4.1 Contribution of This Paper

In this paper, we proposed a model incorporating together SC competition, employee engagement and green investing. To do so, we resorted to GT, the mathematical field of conflict and competition between multiple players. By modelling a static, non-cooperative game between multiple firms producing substitutable products, we managed to capture oligopolistic competition where each firm seeks to maximise its own profits. To the best of our knowledge, for the first time in SCM literature, we combined the still nascent modelling of labour productivity with the modelling of sustainable investments and emissions tracking. The model can provide SC decision makers with:

- the optimal amount of products to produce at different manufacturing facilities in the SCN, and the selection of manufacturing technologies and processes to utilise at these facilities;
- the mode of transport that should be used to deliver the products to distribution centres, which centres should be utilised and in what proportions;
- the choice of technologies to be used for storing the products;
- the selection of means to eventually distribute the products to the target markets;
- the method for optimal investment in green technologies and initiatives throughout the SC according to a predetermined total budget, the specified maximum budget per unit flow on each link, and in case of capital-intensive links, the minimum investment amounts per unit flow that would be required to set up new green links;
- the amount of emissions generated at each link in

the SC, the resultant carbon tax owed, and the amount that will be saved through green investments;

- the labour hours required on each link to achieve such operations, as well as the impact that the engagement of employees with green investments has on productivity; and, finally,
- the optimal profit generated when considering all the above elements together.

Moreover, the model can be used in order to investigate the effects of changes in the SC, be it changes in cost functions, the introduction of new nodes into the SCN or the deletion of nodes, changes in consumers' attitudes with regards to sustainability, changes in the competitors' outputs, increases/decreases in employee engagement levels, and the addition/deletion of demand markets. The model can also be used to conduct sensitivity analysis on a number of different parameters. Finally, the model can be applied by policymakers who would like to study the impact of the introduction of a carbon tax on a specific industry, the effects of international competition and/or the effects of a merger in a specific industry on the resulting competitiveness.

In order to demonstrate the utility of the model in practice, we considered three scenarios inspired by the Maltese dairy industry. Through these scenarios, a number of managerial insights emerged. We could see how having a more environmentally conscious competitor in the market can have an impact on the demands, prices and eventual profits of competitors. Through the sensitivity analysis on the budget parameter, it was shown that one way to recover the market position would be through increased investments in green initiatives, to render the firm more attractive to consumers, to boost the engagement and productivity of employees and ultimately to reduce the harm to the environment. This highlights the impact that SC competition can have on firms, such that one firm opting to prioritise sustainability in its SC could have the domino effect of causing other firms to follow suit.

A key finding of this paper is the importance of employee engagement and well-being within the SC. In Scenario 2, we studied a situation where there was no employee engagement with the green investments being made. Results showed a decline in profitability of both farms, with Farm 2 feeling the effects even stronger due to its considerable investments in building a sustainable SC. Without the engagement of employees, the benefits of the investments could not be enjoyed to their full potential. This highlights how essential employee engagement is in the shift towards a greener SC, thus meriting more research to be focused on this area. Sensitivity analysis on the employee engagement parameter also highlighted that if a firm is perceived as not doing enough (by not investing enough in green initiatives) by its employees, then this will have a negative impact on its profits. Finally, the labour disruption scenario, presented in Scenario 3, highlighted how essential the health of employees is, a theme that was also emphasised in [18]. Thus, as the pandemic recedes into the background, the responsibility now falls on individual firms to ensure that employee health and well-being is prioritised.

4.2 Future Research

Currently, the model is in the form of a static, non-cooperative game, where decisions are made once and simultaneously by all competing firms. An interesting extension to this would be the modelling of a multi-period dynamic game, where decisions are taken multiple times over a number of successive time periods, as in [27, 31]. In our model, we also assumed that all parameters are common knowledge amongst all firms. Thus, future research could include asymmetric information or utilise stochastic programming to model the uncertainty in various parameters. Furthermore, more sophisticated solution algorithms, such as metaheuristic methods [15], may need to be considered when applying the model to more complex SCNs.

With regards to the sustainability aspect, the model could be expanded to consider progressive taxes and emissions penalty systems as in [30], as well as government subsidies for investment as in [31]. With regards to the labour element, future research could implement Lagrange analysis of the variable bounds, as well as the inclusion of elastic wages such that the availability of labour is dependent on the level of wages paid by the firm, as in [17, 19]. Competition between firms for labour could also be modelled through shared labour bounds for all firms, as in [18].

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